Int. J. Heat Mass Transfer. Vol. 30, No. 5, pp. 959–965, 1987
Printed in Great Britain

0017-9310/87 \$3.00 + 0.00
Pergamon Journals Ltd.

Separation of variables solution for non-linear radiative cooling

ROBERT SIEGEL
NASA–Lewis Research Center, Office of the Chief Scientist, 21000 Brookpark Road, Cleveland, OH 44135, U.S.A.

(*Received* 14 *April* 1986 *and in final form* 22 *July* 1986)

Abstract—A separation of variables solution has been obtained for transient radiative cooling of an absorbing–scattering plane layer. The solution applies after an initial transient period required for adjustment of the temperature and scattering source function distributions. The layer emittance, equal to the instantaneous heat loss divided by the fourth power of the instantaneous mean temperature, becomes constant. This emittance is a function of only the optical thickness of the layer and the scattering albedo; its behavior as a function of these quantities is considerably different than for a layer at constant temperature.

INTRODUCTION

THE PURPOSE of this paper is to derive a separation of variables type of solution that arises in the transient radiative cooling of a layer of absorbing and scattering material. The radiative cooling of the layer is governed by a transient non-linear integro-differential equation. The solution yields a constant layer emittance based on the instantaneous heat loss and the instantaneous mean temperature of the layer. The emittance is constant even though the heat loss and mean temperature are both changing with time.

The present study arose in connection with an analysis [1] of the radiative cooling of a layer filled with hot liquid droplets. The use of multiple liquid droplet streams passing in a directed manner through space, has been proposed as a waste heat dissipation technique for an orbiting space power plant [2]. Detailed discussions of the system components and the system optimization are given in refs. [3, 4]. The droplet layer may consist of many thousands of individual streams with possible droplet diameters being 50–200 μm. An analysis of the transient cooling of the droplet layer was carried out in detail in ref. [1] by treating the layer as an absorbing, emitting and scattering medium. The transient energy equation of radiative transfer was solved numerically. This expanded on previous work such as refs. [2, 5–7]. Additional general information on transient radiative cooling is given in refs. [8, 9]. During the transient solution in ref. [1], an instantaneous emittance was calculated by dividing the instantaneous heat loss from one side of the layer by the fourth power of the mean temperature of the layer. The cooling started from a constant initial temperature across the layer. During the cooling, a temperature distribution develops and then diminishes in amplitude as energy is lost; typical temperature distributions are given in refs. [1, 6]. It was found that, after the layer had lost approximately 30% of its energy (see ref. [1] for more precise results), the emittance became a constant that depended only on the optical thickness and scattering albedo of the layer.

Initially the layer has an emittance corresponding to the initial uniform temperature distribution. As the transient proceeds, the emittance decreases since the outer regions of the layer become cool relative to the mean temperature. For each set of parameters, the emittance leveled out at a constant value below the initial emittance. An examination of the transient distributions in temperature and the one-fourth power of the source function showed that they had certain time-independent characteristics. These findings indicated that a solution could be found in the form given in this paper. The results yield a simple cooling relation that applies following the initial transient period.

ANALYSIS

The concept of the liquid-drop radiator for dissipation of waste heat for space applications is shown schematically in Fig. 1. The droplet-filled layer of thickness D is moving at uniform velocity $\bar{u}$ as it cools by radiation. Instead of calculating the heat transfer behavior as a function of distance z, a transformation is used in terms of the time after initial exposure to the cold environment, $\tau = z/\bar{u}$. The analysis can then be carried out as the transient cooling of a plane layer exposed to an environment at $T_e = 0$. The radiative properties are assumed gray, which is a reasonable approximation for the droplet material properties shown in ref. [3] for the infra-red region. Isotropic scattering is used; this should yield satisfactory results as shown in ref. [10], where the effect of anisotropic

NOMENCLATURE

a absorption coefficient of absorbing–scattering layer
c_p specific heat of absorbing–scattering droplet-filled layer
D thickness of absorbing–scattering layer
E_1, E_2 exponential integral functions,
$$E_n(x) = \int_0^1 \mu^{n-2} \exp(-x/\mu)\,d\mu$$
$F(X)$ function of X in temperature profile
F_m integrated mean value of $F(X)$
$G(X)$ function of X in profile of $\tilde{I}^{1/4}$
$f(X)$ universal temperature distribution
$g(X)$ universal distribution of $I^{1/4}$
I source function in absorbing–scattering layer; $\tilde{I} = \pi I/\sigma T_i^4$
q heat loss per unit area and time from one side of the radiating layer
q_r radiative heat flow per unit area and time
T absolute temperature, $\tilde{T}T_i$
T_e temperature of surrounding environment
T_i initial temperature of radiating layer
T_m integrated mean temperature across layer at any time during cooling transient, $\tilde{T}_m T_i$
$\bar{u}$ velocity through space of the droplet-filled layer
x, z coordinates across and along the layer
X dimensionless variable, x/D
X^* dummy variable of integration.

Greek symbols
$\varepsilon_{m,s}$ steady value of layer emittance (based on instantaneous value of T_m) achieved after initial transient
κ optical coordinate, $(a+\sigma_s)x$; κ_D optical thickness of layer, $(a+\sigma_s)D$
κ^* dummy variable of integration
ρ density of droplet-filled layer
σ Stefan–Boltzmann constant
σ_s scattering coefficient of droplet-filled layer
τ time from start of cooling, $\tilde{\tau}/(4\sigma T_i^3/\rho c_p D)$
Ω albedo for scattering, $\sigma_s/(a+\sigma_s)$.

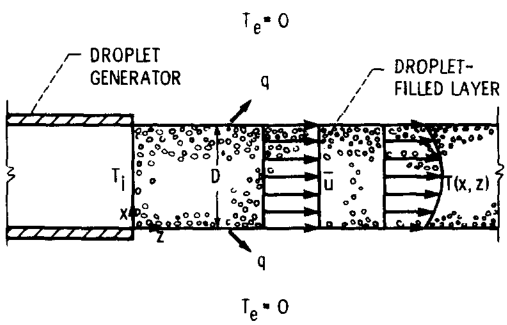

FIG. 1. Geometry of radiating layer filled with hot droplets.

scattering was examined for a cloud of particles and compared with isotropic results.

The basic equations for transient cooling by radiation of an absorbing–scattering layer are given by the energy equation

$$\frac{\rho c_p}{a+\sigma_s}\frac{\partial T}{\partial \tau} = -\frac{\partial q_r}{\partial \kappa} \qquad T(\kappa, \tau = 0) = T_i \tag{1}$$

and the relations for the radiative flux derivative and the source function

$$-\frac{\partial q_r}{\partial \kappa} = 2\pi \int_0^{\kappa_D} I(\kappa^*, \tau)E_1(|\kappa - \kappa^*|)\,d\kappa^* - 4\pi I(\kappa, \tau) \tag{2}$$

$$I(\kappa, \tau) = (1-\Omega)\frac{\sigma T^4(\kappa, \tau)}{\pi} + \frac{\Omega}{2}\int_0^{\kappa_D} I(\kappa^*, \tau)E_1(|\kappa-\kappa^*|)\,d\kappa^*. \tag{3}$$

By eliminating the radiative flux q_r and placing the results in dimensionless form, the equations become

$$\tilde{I}(X, \tilde{\tau}) = (1-\Omega)\tilde{T}^4(X, \tilde{\tau}) + \frac{\kappa_D \Omega}{2}\int_0^1 \tilde{I}(X^*, \tilde{\tau})E_1(\kappa_D|X - X^*|)\,dX^* \tag{4a}$$

$$\frac{\partial \tilde{T}}{\partial \tilde{\tau}} = \kappa_D \frac{(1-\Omega)}{\Omega}[\tilde{I}(X, \tilde{\tau}) - \tilde{T}^4(X, \tilde{\tau})]. \tag{4b}$$

The previous transient numerical solution [1] revealed that both the temperature distribution and the distribution of the source function to the one-quarter power, adjusted to shapes that became independent of time so that

$$\frac{T(\kappa, \tau) - T(0, \tau)}{T(\kappa_D/2, \tau) - T(0, \tau)} = f(X) \tag{5a}$$

$$\frac{I^{1/4}(\kappa, \tau) - I^{1/4}(0, \tau)}{I^{1/4}(\kappa_D/2, \tau) - I^{1/4}(0, \tau)} = g(X). \tag{5b}$$

Then

$$T(\kappa,\tau)/T(0,\tau) = 1+\{[T(\kappa_D/2,\tau) - T(0,\tau)]/T(0,\tau)\}f(X)$$

and similarly for $I^{1/4}(\kappa,\tau)$. A further examination of the transient numerical solution showed that, after an initial transient period, the quantity $[T(\kappa_D/2,\tau)-T(0,\tau)]/T(0,\tau)$ becomes constant, and the similar quantity for $I^{1/4}$ is also a constant.

The previous results lead to the idea of trying to obtain a solution to equations (4a) and (4b) in the separated form

$$\tilde{T}(X,\tilde{\tau}) = \tilde{T}(0,\tilde{\tau})F(X) \quad (6a)$$

$$\tilde{I}^{1/4}(X,\tilde{\tau}) = \tilde{T}(0,\tilde{\tau})G(X). \quad (6b)$$

If the equations can be satisfied, this will be a valid solution. Substituting equations (6a) and (6b) into equations (4a) and (4b) gives

$$G^4(X) = (1-\Omega)F^4(X)+\frac{\kappa_D\Omega}{2} \times \int_0^1 G^4(X^*)E_1(\kappa_D|X-X^*|)\,\mathrm{d}X^* \quad (7a)$$

$$\frac{1}{\tilde{T}^4(0,\tilde{\tau})}\frac{\mathrm{d}\tilde{T}(0,\tilde{\tau})}{\mathrm{d}\tilde{\tau}} = \kappa_D\frac{(1-\Omega)}{\Omega}\frac{G^4(X)-F^4(X)}{F(X)}. \quad (7b)$$

In equation (7b) the variables in $\tilde{\tau}$ and X have been separated; hence, if the proposed solution is valid, each side of this equation must be a constant. Then the function of X on the right-hand side of equation (7b) can be replaced by the value at any X. Since $F(0) = 1$, it is convenient to use the relation that

$$[G^4(X)-F^4(X)]/F(X) = G^4(0)-1.$$

This is rearranged into the form

$$F^4(X) = G^4(X)+[1-G^4(0)]F(X). \quad (8)$$

As described later, this was solved simultaneously with equation (7a) to obtain $F(X)$ and $G(X)$.

The mean temperature of the layer at any time is

$$\tilde{T}_{\mathrm{m}}(\tilde{\tau}) = \int_0^1 \tilde{T}(X,\tilde{\tau})\,\mathrm{d}X = \tilde{T}(0,\tilde{\tau})\int_0^1 F(X)\,\mathrm{d}X = \tilde{T}(0,\tilde{\tau})F_{\mathrm{m}}.$$

Then

$$\frac{1}{\tilde{T}^4(0,\tilde{\tau})}\frac{\mathrm{d}\tilde{T}(0,\tilde{\tau})}{\mathrm{d}\tilde{\tau}} = F_{\mathrm{m}}^3\frac{1}{\tilde{T}_{\mathrm{m}}^4}\frac{\mathrm{d}\tilde{T}_{\mathrm{m}}}{\mathrm{d}\tilde{\tau}}. \quad (9)$$

A heat balance on the layer gives

$$2q = 2\varepsilon_{\mathrm{m,s}}\sigma T_{\mathrm{m}}^4(\tau) = -\rho c_p D\frac{\mathrm{d}T_{\mathrm{m}}(\tau)}{\mathrm{d}\tau}$$

where $\varepsilon_{\mathrm{m,s}}$ is the steady value of emittance that is achieved based on the instantaneous mean temperature. In dimensionless form this becomes

$$\frac{1}{\tilde{T}_{\mathrm{m}}^4(\tilde{\tau})}\frac{\mathrm{d}\tilde{T}_{\mathrm{m}}(\tilde{\tau})}{\mathrm{d}\tilde{\tau}} = -\frac{\varepsilon_{\mathrm{m,s}}}{2}. \quad (10)$$

By combining equations (10) and (9) to eliminate $\tilde{T}_{\mathrm{m}}$ and then using equations (7b) and (8), the emittance is obtained in terms of F and G

$$\varepsilon_{\mathrm{m,s}} = \frac{2}{F_{\mathrm{m}}^3}\frac{\kappa_D(1-\Omega)}{\Omega}[1-G^4(0)]. \quad (11)$$

After $F(X)$ and $G(X)$ are obtained, the universal profiles $f(X)$ and $g(X)$ can be obtained by combining equations (5) and (6) to obtain

$$f(X) = \frac{F(X)-F(0)}{F(1/2)-F(0)} \quad (12a)$$

$$g(X) = \frac{G(X)-G(0)}{G(1/2)-G(0)}. \quad (12b)$$

Equations (7a) and (8) are not convenient to solve for $F(X)$ when there is only absorption in the layer, $\Omega = 0$. In this instance, $\tilde{I}(\kappa,\tau) = \tilde{T}^4(\kappa,\tau)$, and the original equations (1)–(3) reduce to

$$\frac{\partial\tilde{T}}{\partial\tilde{\tau}} = -\kappa_D\left(\tilde{T}^4(X,\tilde{\tau})-\frac{\kappa_D}{2} \times \int_0^1 \tilde{T}^4(X^*,\tilde{\tau})E_1(\kappa_D|X-X^*|)\,\mathrm{d}X^*\right). \quad (13)$$

Equation (6a) is substituted for $\tilde{T}$ to yield

$$\frac{1}{\tilde{T}^4(0,\tilde{\tau})}\frac{\mathrm{d}\tilde{T}(0,\tilde{\tau})}{\mathrm{d}\tilde{\tau}} = -\frac{\kappa_D}{F(X)}\left(F^4(X)-\frac{\kappa_D}{2} \times \int_0^1 F^4(X^*)E_1(\kappa_D|X-X^*|)\,\mathrm{d}X^*\right). \quad (14)$$

Since the variables are separated, each side of equation (14) must be a constant. This is conveniently evaluated by using the right-hand side and letting $X = 0$ (note that $F(0) = 1$). Then this constant is equated to the right-hand side of equation (14) and the result rearranged into the integral equation that was solved numerically for $F(X)$

$$F^4(X) = F(X)\left(1-\frac{\kappa_D}{2}\int_0^1 F^4(X^*)E_1(\kappa_D|0-X^*|)\mathrm{d}X^*\right) + \frac{\kappa_D}{2}\int_0^1 F^4(X^*)E_1(\kappa_D|X-X^*|)\,\mathrm{d}X^*. \quad (15)$$

From equations (9) and (10), the left-hand side of equation (14) is equal to $-F_{\mathrm{m}}^3\varepsilon_{\mathrm{m,s}}/2$. This is then equated to the right-hand side of equation (14) evaluated at $X = 0$ to yield an equation for $\varepsilon_{\mathrm{m,s}}$ that is evaluated after $F(X)$ is obtained from equation (15)

$$\varepsilon_{\mathrm{m,s}} = \frac{2\kappa_D}{F_{\mathrm{m}}^3}\left(1-\frac{\kappa_D}{2}\int_0^1 F^4(X^*)E_1(\kappa_D|0-X^*|)\,\mathrm{d}X^*\right). \quad (16)$$

Equations (7a) and (8) were solved numerically to yield $F(X)$ and $G(X)$. The first several cases were solved using $F(X) = 1$ and $G(X) = 1$ as initial guesses. The initial functions were inserted into the right-hand side of equation (7a). The difference between the right-hand side and trial $G^4(X)$ was multiplied by an acceleration factor (usually 1.2) and the result added to the trial $G^4(X)$ to obtain a new $G^4(X)$ with which to continue the iteration of equation (7a). After convergence within a small tolerance (usually 10^{-4}), an $F^4(X)$ was evaluated from equation (8) by using the old $F(X)$ on the right-hand side. An acceleration factor (usually 1.5) was then applied to this $F^4(X)$ to obtain a new $F^4(X)$ with which to repeat the iteration of equation (7a). The calculations were checked by reducing the size of ΔX; usually $40\Delta X$ intervals were used across the layer. After convergence of the $F(X)$ and $G(X)$, the emittance was obtained from equation (11) in which F_m was found by integrating $F(X)$ across the layer. After some results were obtained, the calculations were started from approximate $F(X)$ and $G(X)$ functions. This decreased the number of iterations required which was especially helpful as the κ_D increased in value. A larger κ_D increased the sensitivity of equation (7a) and more iterations were required to obtain $G^4(X)$ within a given error tolerance.

For absorption only in the layer ($\Omega = 0$), equation (15) was iterated by putting a trial $F(X)$ into the right-hand side and applying an acceleration factor of 1.2 to the resulting $F^4(X)$ to obtain the next approximation. After convergence of $F^4(X)$ to at least four decimal places for all X, the emittance was evaluated from equation (16).

The solution procedure requires an accurate integration technique, and since $E_1(0) = \infty$, special consideration is needed as X^* approaches X. Since the integral of E_1 is $-E_2$, and $E_2(0) = 1$, the integration was performed analytically for a small region adjacent to the singularity, with F and G each held constant over this small region. The calculations were checked by reducing the size of this region to be sure its size had no effect. The integrations in the regions away from the singularity were performed with a Gaussian integration subroutine available in the computer library. Solutions generally required only a few minutes on an IBM 370 computer. A few cases with $\kappa_D = 14$ required about 6 min of time as a result of slower convergence of the iterative solution of equation (7a).

Since $\varepsilon_{m,s}$ is independent of time, the cooling equation (10) can be integrated from time $\tilde{\tau}_1$ and $\tilde{\tau}_2$ to yield

$$\Delta\tilde{\tau} = \tilde{\tau}_2 - \tilde{\tau}_1 = \frac{2}{3\varepsilon_{m,s}}\left(\frac{1}{\tilde{T}^3_{m,2}} - \frac{1}{\tilde{T}^3_{m,1}}\right). \qquad (17)$$

As shown by the results in ref. [1], the $\varepsilon_{m,s}$ applies after approximately 30% of the energy has been lost from the layer. Hence, starting with any $\tilde{T}_{m,1}$ less than about 0.7, the time required to reach any lower $\tilde{T}$ can be easily obtained by use of equation (17).

RESULTS AND DISCUSSION

As described in the analysis for cases with $\Omega > 0$, equations (7a) and (8) were solved to obtain $F(X)$ and $G(X)$, and equation (15) was solved for $F(X)$ when $\Omega = 0$. This was done for a range of κ_D and Ω values. The $F(X)$ and $G(X)$ were in agreement with those obtained in the transient solution [1]. The right-hand sides of equations (7b) and (14) were checked to verify that they were independent of X. The values of $\varepsilon_{m,s}$ were obtained from equations (11) and (16), and were found to be in agreement with the values available from ref. [1].

With the separation of variables solution thus verified, a wide range of $\varepsilon_{m,s}$ values were calculated; they are given in Table 1 and are plotted in Fig. 2. In Fig. 2(a) results are given as a function of κ_D for various Ω. Consider first the curve for $\Omega = 0$. As the optical thickness is increased from zero, the emittance increases with κ_D and reaches a maximum at κ_D a little above 2. For larger κ_D, the effect of the outer regions of the radiating layer being cool becomes increasingly important and, consequently, the $\varepsilon_{m,s}$ decreases as κ_D is further increased. This is in contrast to the results for an emitting layer at uniform temperature. In this instance the emittance increases monotonically toward unity as κ_D is increased.

For $\Omega > 0$ the curves in Fig. 2(a) have the same general trend as the curve for $\Omega = 0$. Increased scattering corresponds to lower absorption, and hence emission, within the layer. Hence, for a fixed κ_D the $\varepsilon_{m,s}$ values decrease as Ω is increased. The location of the maximum $\varepsilon_{m,s}$ for a fixed Ω shifts to larger κ_D values as Ω is increased.

The $\varepsilon_{m,s}$ are plotted in Fig. 2(b) as a function of Ω for constant values of κ_D from 0.5 to 14. For a layer that has a fairly small optical thickness, $\kappa_D = 0.5$, the $\varepsilon_{m,s}$ decreases almost linearly as Ω is increased. In the range of $\kappa_D = 1$ and 2, the $\varepsilon_{m,s}$ is increasing with κ_D, and, compared with results at smaller κ_D, the $\varepsilon_{m,s}$ is decreasing somewhat more slowly with Ω in the region of small Ω. For optically thick layers with large κ_D, such as 10 or 14, the $\varepsilon_{m,s}$ is almost constant with Ω until Ω reaches about 0.8; then there is a rapid decrease of $\varepsilon_{m,s}$ toward zero as Ω is further increased. This behavior is considerably different than the emittance results for a layer at constant temperature as given in refs. [1, 2].

Figure 3 gives the temperature distribution during the 'fully developed' portion of the cooling transient analyzed here. The $f(X)$ defined by equation (5a) is given in Fig. 3(a) for three different κ_D values and no scattering, $\Omega = 0$. The profiles are quite insensitive to the value of the optical thickness. Figure 3(b) shows the variation of $f(X)$ with Ω for a fixed thickness, $\kappa_D = 7$. This reveals that the $f(X)$ is also insensitive to the amount of scattering present. Thus $f(X)$ is

Table 1. Values of layer emittance, $\varepsilon_{m,s}(\kappa_D, \Omega)$

Optical thickness, κ_D	Scattering albedo, Ω						
	0	0.30	0.60	0.80	0.90	0.95	0.98
1	0.772	0.662	0.489	0.304	0.173	0.093	0.039
2	0.894	0.816	0.669	0.472	0.297	0.170	0.075
3	0.882	0.830	0.722	0.555	0.379	0.232	0.107
5	0.777	0.753	0.696	0.592	0.456	0.313	0.161
7	0.672	0.659	0.627	0.563	0.468	0.351	0.200
10	0.551	0.544	0.529	0.496	0.440	0.360	0.233
14	0.440	0.437	0.430	0.414	0.385	0.338	0.248

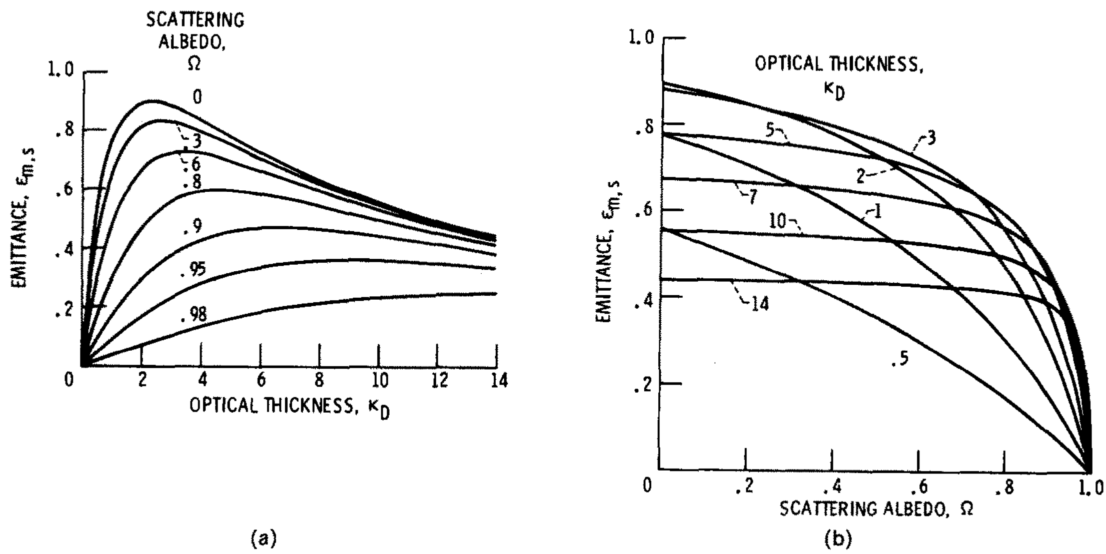

FIG. 2. Emittance of absorbing–scattering layer in 'fully developed' region: (a) emittance as a function of optical thickness; (b) emittance as a function of scattering albedo.

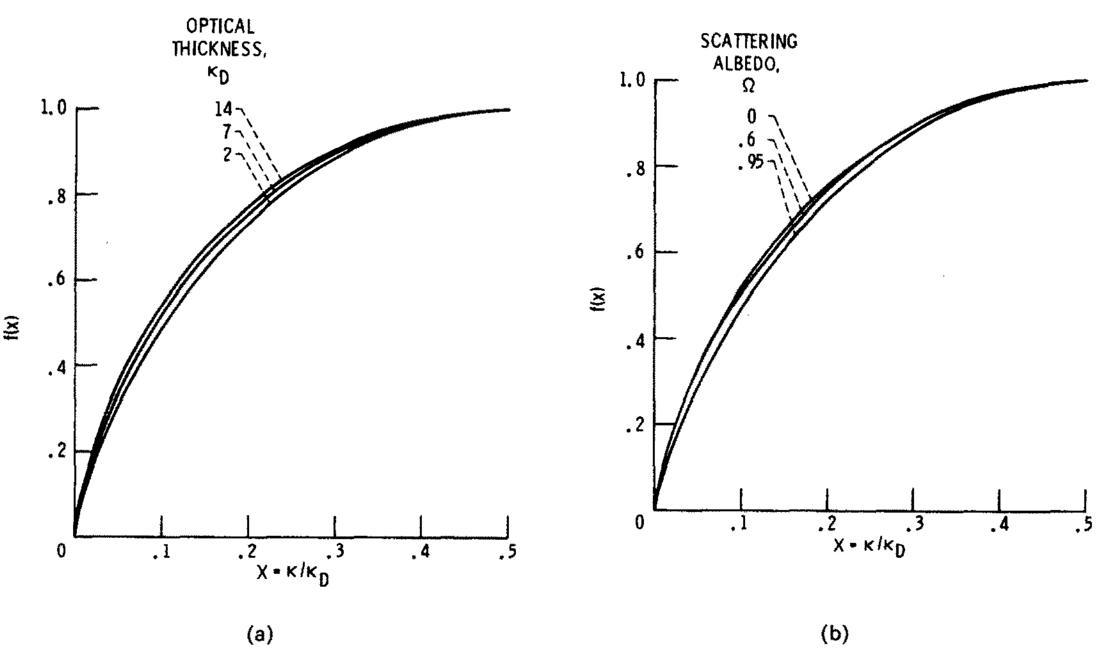

FIG. 3. Dimensionless temperature function: (a) scattering albedo, $\Omega = 0$; (b) optical thickness, $\kappa_D = 7$.

almost a universal function, as it does not vary appreciably with either κ_D or Ω for the ranges studied here.

In Fig. 4 the $g(X)$ is given for various κ_D with $\Omega = 0.6$; $g(X)$ is the dimensionless profile of the one-quarter power of the source function as defined in equation (5b). These functions were also found to be quite insensitive to both κ_D and Ω, and $g(X)$ has a shape very close to the shape of $f(X)$. Because the variation of $g(X)$ with Ω is quite small, the results for $g(X)$ in Fig. 4 for $\Omega = 0.6$ are very close to the curves for $f(X)$ in Fig. 3(a) which are for $\Omega = 0$ (recall that $\tilde{I}^{1/4} \to \tilde{T}$ as $\Omega \to 0$). For each κ_D the $g(X)$ for all Ω were found to be very close to the $f(X)$ for $\Omega = 0$.

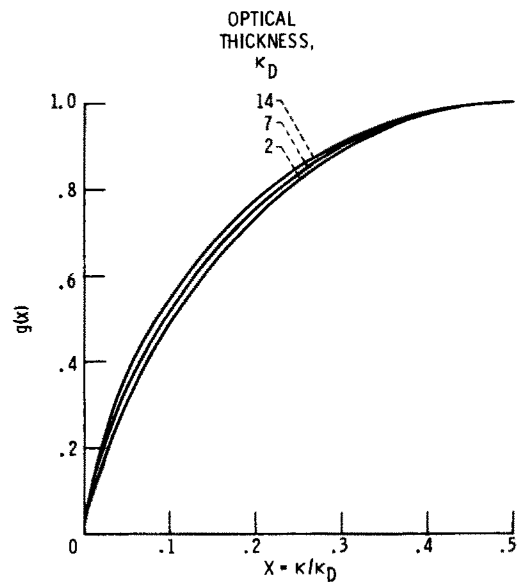

FIG. 4. Dimensionless form of one-quarter power of source function as a function of optical thickness with $\Omega = 0.6$.

Using the $\varepsilon_{m,s}$ values from Fig. 2, cooling times during the 'fully developed' transient period are readily calculated from equation (17). As shown by the mean temperature results in Table 2 of ref. [1], this can be used after the emitting layer has lost approximately 30% of its energy.

REFERENCES

1. R. Siegel, Transient radiative cooling of a droplet-filled layer, *J. Heat Transfer* **109**, 159–164 (1987).
2. A. T. Mattick and A. Hertzberg, Liquid droplet radiators for heat rejection in space, *J. Energy* **5**, 387–393 (1981).
3. A. F. Presler, C. E. Coles, P. S. Diem-Kirsop and K. A. White, Liquid Droplet Radiator Program at the NASA Lewis Research Center, ASME paper 86-HT-15, AIAA/ASME Thermophysics and Heat Transfer Conference, Boston, Massachusetts (June 1986).
4. R. T. Taussig and A. T. Mattick, Droplet radiator systems for spacecraft thermal control, *J. Spacecraft Rockets* **23**, 10–17 (1986).
5. P. S. Bathla and R. Viskanta, Effect of surroundings on the transient energy transfer in a layer of radiating gas, *Appl. Scient. Res.* **19**, 182–197 (1968).
6. R. Viskanta and P. S. Bathla, Unsteady energy transfer in a layer of gray gas by thermal radiation, *Z. Angew. Math. Phys.* **18**, 353–367 (1967).
7. S. Kubo, Unsteady radiative heat transfer in a scattering-dominant medium, *AIAA J.* **22**, 1804–1809 (1984).
8. R. Viskanta and E. E. Anderson, Heat transfer in semi-transparent solids. In *Advances in Heat Transfer* (Edited by T. F. Irvine, Jr. and J. P. Hartnett), Vol. 11, pp. 317–441. Academic Press, New York (1975).
9. R. Siegel and J. R. Howell, *Thermal Radiation Heat Transfer*, 2nd Edn. Hemisphere, Washington, D.C. (1981).
10. L. W. Stockham and T. J. Love, Radiative heat transfer from a cylindrical cloud of particles, *AIAA J.* **6**, 1935–1940 (1968).

SOLUTION AVEC SEPARATION DES VARIABLES POUR LE REFROIDISSEMENT RADIATIF NON LINEAIRE

Résumé—Une solution par séparation des variables a été obtenue pour le refroidissement radiatif variable d'une couche plane absorbante et diffusante, La solution s'obtient après une péroide initiale transitoire nécessaire pour l'ajustement des distributions de température et de la diffusion. L'émittance de la couche, égale à la perte de chaleur instantanée divisée par la puissance quatrième de la température moyenne instantanée, devient constante. Cette émittance est fonction seulement de l'épaisseur optique de la couche et de l'albédo; son comportement en fonction de ces paramètres est considérablement différent de celui pour une couche à température constante.

EINE LÖSUNG DURCH TRENNUNG DER VARIABLEN FÜR NICHTLINEARE STRAHLUNGSKÜHLUNG

Zusammenfassung—Für die instationäre Strahlungskühlung einer streuend-absorbierenden, ebenen Schicht wurde eine Lösung durch Trennung der Variablen erhalten. Die Lösung gilt nach einer anfänglichen instationären Periode, die für das Erreichen der stationären Temperatur- und Streufunktionsverteilungen notwendig ist. Die Emission der Schicht, die gleich dem Quotient aus dem momentanen Wärmeverlust under der vierten Potenz der augenblicklichen Temperatur ist, wird konstant. Diese Emission ist nur von der optischen Dicke der Schicht und der Streuungsalbedo abhängig. Das Emissionsverhalten als Funktion dieser Größen unterscheidet sich beträchtlich von dem einer Schicht mit konstanter Temperatur.

РЕШЕНИЕ МЕТОДОМ РАЗДЕЛЕНИЯ ПЕРЕМЕННЫХ ЗАДАЧИ НЕЛИНЕЙНОГО РАДИАЦИОННОГО ОХЛАЖДЕНИЯ

Аннотация—Методом разделения переменных получено решение задачи нестационарного радиационного охлаждения поглощающего–рассеивающего плоского слоя. Решение справедливо после начального переходного периода, необходимого для согласования распределения температуры и функции источника рассеяния. Излучательная способность слоя, равная мгновенной потере тепла, отнесенной к мгновенной средней температуре в четвертой степени, становится постоянной. Эта излучательная способность является функцией только оптической толщины слоя и альбедо рассеяния; ее поведение как функции этих величин существенно отличается от поведения для слоя постоянной температуры.